The Scheme Programming Language (continued)

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CS 152 – Programming Paradigms

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Programming languages that are part of the Lisp family have foundational support for lists.

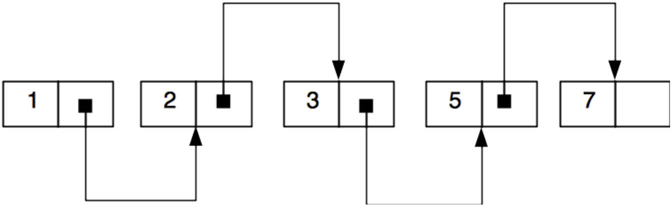
List are implemented as linked lists.

List: '(1 2 3 5 7)

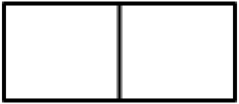
Dot Notation: '(1 . (2 . (3 . (5 . (7 . '()))))))

cons List: (cons 1 (cons 2 (cons 3 (cons 5 (cons 7 '())))))

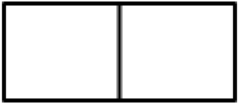
Diagram:



cons Cell

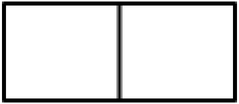


cons Cell



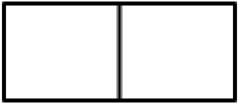
- We can access the left side of the cons cell using the `car` function.

cons Cell



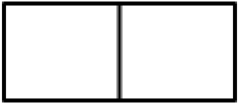
- We can access the left side of the cons cell using the `car` function.
- We can access the right side of the cons cell using the `cdr` functions.

cons Cell



- We can access the left side of the cons cell using the `car` function.
- We can access the right side of the cons cell using the `cdr` functions.
- The names `car` and `cdr` are historical hardware references dating back to the 1950s.

cons Cell



- cons cells can be used to store pairs of objects.
  - Example: `(cons 2 3) => '(2 . 3)` (note the dot syntax)
- cons cells are more often used for making lists
  - First element is typically a value, while the second element is either another cons cell or an empty list `'()`.
  - Example: `(cons 1 (cons 2 (cons 3 '()))) => '(1 2 3)`
  - `(first '(1 2 3)) => 1`
  - `(rest '(1 2 3)) => '(2 3)`

`first` and `rest` vs. `car` and `cdr`

- Use `first` and `rest` for lists.
- In Racket, you must use `car` and `cdr` for non-list cons cells.

S-expressions

An S-expression is one of the following:

- A primitive (e.g., a number, a string, a symbol)
- A list without dots (e.g., `(1 3 5)` and `(+ x y)`)
- A list in dot notation (e.g., `'(1 . (3 . (5 . ())))`)
- Special dot notation case: `(1 3 5 . 7)`
  - Equivalent to `(1 . (3 . (5 . 7)))`

A Lisp program is a collection of S-expressions.

What is the difference between `(+ 2 4)` and `'(+ 2 4)`

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`(+ 2 4)` is interpreted to call the function + with arguments 2 and 4. In the case of `'(+ 2 4)`, the `'` tells the interpeter not to evaluate what's after it, and so it returns the list `(+ 2 4)` unevaluated.

Quoting

' is shorthand for the `quote` function.

Example: `'x` is equivalent to `(quote x)`.
`'(1 3 5)` is the equivalent of `(quote (1 3 5))`.

Operations on Lists

- `cons` can easily be used to add to the front of a list, returning a new list.
  - Example: `(cons 1 '(2 3)) => '(1 2 3)`
- Use the `append` function to concatenate two lists.
  - Example: `(append '(1 2) '(3 4)) => '(1 2 3 4)`
- Use the `length` function to get the length of the list.
- Use the `empty?` function to check if the list is empty.
- `reverse` reverses a list.

Iterating Over Lists

Since we are coding in functional programming style, we need to use recursion. Thankfully, `first` and `rest` make it easy to write list-traversal code in an easy-to-use manner.

Iterating Over Lists

Since we are coding in functional programming style, we need to use recursion. Thankfully, `first` and `rest` make it easy to write list-traversal code in an easy-to-use manner.

```
; Return the sum of the elements in elem
(define (element-sum elems sum)
  (if (empty? elems)
      sum
      (element-sum (rest elems) (+ sum (first elems)))))
```

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Anonymous Functions with `lambda`

Definition (`lambda`)

`(lambda (arg1 ... argN) function-body)` creates an anonymous function with arguments `arg1` to `argN` that evaluates `function-body`.

Example of Usage of Anonymous Functions

Remember the sort comparison I gave during the last function? Here is an example of using one

```
(sort '(3 2 5 4)
      (lambda (a b)
        (cond ((< a b) -1)
              ((> a b) 1)
              (else 0))))
```

We are passing a custom comparison function that sort can use to compare two numbers. (Note that this is a custom sort function, not Racket's built-in `sort`.)

It turns out that

```
(define (function-name arg1 ...  argN) function-body)
```

is simply syntactic sugar for

```
(define function-name
        (lambda (arg1 ... argN) function-body))
```

In order words, we assign an anonymous function a name.

```
(define (normal-compare a b)
  (cond ((< a b) -1)
        ((> a b) 1)
        (else 0)))

(sort '(3 2 5 4) normal-compare)
```

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Technically we could define variables using `lambda` by declaring unused function arguments:

```
; Computes the hypotenuse z = sqrt(x^2 + y^2)
(define (hypotenuse x y)
  ((lambda (xs ys)
     (expt (+ xs ys) 0.5))
   (expt x 2) (expt y 2)))
```

`xs` is set to the result of `(expt x 2)`, and `ys` is set to the result of `(expt y 2)`.

Scheme provides syntactic sugar for defining local variables. We can use `let` to define local variables.

Definition (`let`)

`(let ((var1 def1) ... (varN defN)) expr)` defines the variables `var1 = def1` to `varN = defN`, where `def1` to `defN` are expressions. `expr` is able to use these variables.

```
; Computes the hypotenuse z = sqrt(x^2 + y^2)
(define (hypotenuse x y)
  (let ((xs (expt x 2))
        (ys (expt y 2)))
    (expt (+ xs ys) 0.5)))
```

What is wrong with this code?

```
; double the numbers in a list
(define (double-list elems)
  (if (empty? elems)
      '()
      (let ((elem (first elems))
            (double-elem (* elem 2)))
        (cons double-elem (double-list (rest elems))))))
```

What is wrong with this code?

```
; double the numbers in a list
(define (double-list elems)
  (if (empty? elems)
      '()
      (let ((elem (first elems))
            (double-elem (* elem 2)))
        (cons double-elem (double-list (rest elems))))))
```

The culprit is in the definition of `double-elem`. The value of `double-elem` is set to `(* elem 2)`. The problem, though, is that we cannot evaluate `elem` because `let` definitions have no access to variables defined by that same `let`.

Here is one possible solution:

```
; double the numbers in a list
(define (double-list elems)
  (if (empty? elems)
      '()
      (let ((elem (first elems)))
        (let ((double-elem (* elem 2)))
          (cons double-elem (double-list (rest elems)))))))
```

We could have a `let` embedded in a `let`, which solves the scoping problem, but this can get unwieldy for long lists of definitions with dependencies.

```
; double the numbers in a list
(define (double-list elems)
  (if (empty? elems)
      '()
      (let* ((elem (first elems))
             (double-elem (* elem 2)))
        (cons double-elem (double-list (rest elems))))))
```

A better solution is `let*`, which allows the definition of variables that depend on previous variable definitions within the same `let*` definition list.

We can set anonymous functions to variables using `let` and `let*`:

```
(define (normal-sort elems)
  (let ((compare (lambda (a b)
                   (cond ((< a b) -1)
                         ((> a b) 1)
                         (else 0)))))
    (sort elems compare)))
```

Be cautious about declaring recursive functions inside of a `let` or a `let*`. The following code does not work:

```
(define (factorial n)
  (let ((fact (lambda (x answer)
                (if (<= x 1)
                    answer
                    (fact (- x 1) (* x answer))))))
    (fact n 1)))
```

It does not work because the lambda has no access to `fact`. To get it to work, we replace `let` or `let*` with `letrec`.

```
(define (factorial n)
  (letrec ((fact (lambda (x answer)
                   (if (<= x 1)
                       answer
                       (fact (- x 1) (* x answer))))))
    (fact n 1)))
```

Summary

- Scheme has a family of functions used for defining local variables: `let`, `let*`, and `letrec`.
- Use `let` when defining local variables that have no dependencies on other local variables defined in the same `let` scope.
- Use `let*` when defining local variables that depend on previously-defined variables within the same `let*` list.
- Use `letrec` when defining recursive local functions using `lambda`.
- When in doubt, use `letrec`, but stylistically use the most restrictive function of the `let` family applicable to your program (e.g., don't use a `let*` when a `let` is appropriate).

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The Downside of Recursion

- In functional programming, recursion is the preferred way of performing repetitive tasks that would normally be implemented as loops in procedural programs.

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- In functional programming, recursion is the preferred way of performing repetitive tasks that would normally be implemented as loops in procedural programs.
- However, deep levels of recursion could result in a stack overflow error, caused by too many function calls on the call stack.
- To solve this problem, we can use tail recursion.

Tail Recursion

Tail recursion is a style of recursion where the final call of a recursive function is a call to itself.

Recursion vs. Tail Recursion

Without tail recursion:

```
(define (factorial n)
  (if (<= n 1) 1 (* n (factorial (- n 1)))))
```

Recursion vs. Tail Recursion

Without tail recursion:

```
(define (factorial n)
  (if (<= n 1) 1 (* n (factorial (- n 1)))))
```

The reason why this is not tail recursive is because when $n \geq 1$, `factorial`'s final (tail) function call is `*`.

With tail recursion

```
(define (factorial n)
  (letrec ((fact (lambda (x answer)
                   (if (<= x 1)
                       answer
                       (fact (- x 1) (* x answer))))))
    (fact n 1)))
```

With tail recursion

```
(define (factorial n)
  (letrec ((fact (lambda (x answer)
                   (if (<= x 1)
                       answer
                       (fact (- x 1) (* x answer))))))
    (fact n 1)))
```

The reason why this is tail recursive is because when $n \geq 1$, `fact`'s final (tail) function call is `fact`.

Another example of tail recursion:

```
(define (my-length elems)
  (letrec ((my-len (lambda (e len)
                     (if (empty? e)
                         len
                         (my-len (rest e) (+ 1 len))))))
    (my-len elems 0)))
```

Why Tail Recursion

- Scheme and many other functional programming language interpreters offer tail call optimization that is able to detect tail recursion and execute them in such a way where it is essentially a `do` loop, thus avoiding the repeated use of the call stack and thus avoiding stack overflow errors.
- Standards-compliant Scheme interpreters *must* implement tail call optimization.
- We'll learn more about the implementation details when we cover virtual machines and compilation in two weeks.

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Functional programming style involves a lot of recursive list traversals.

What if the language provided library functions that facilitated common patterns, such as:

- Performing an operation on all elements of the list (e.g., doubling all the numbers, or making all strings in a list uppercase)
- Removing elements from a list (e.g., removing a list of all non-prime numbers)
- Performing an aggregate operation on a list returning a single element (e.g., adding all numbers in a list)

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We have these features in Scheme: map, filter, and fold (respectively)

Map Function

Definition (map)

`(map function list1 ... listN)` applies `function` to all elements of each list `list1` to `listN`, where each list element serves as an argument to `function`. For example, if there are N lists, then there are N arguments to `function`.

Example

```
; Answer is '(1 4 9 16 25)
(map (lambda (x) (* x x)) '(1 2 3 4 5))
; Answer is '(5 7 9)
(map + '(1 2 3) '(4 5 6))
```

Filter Function

Definition (filter)

`(filter function list)` applies `function` (with one argument) to each element of `list`, constructing a new list where `function` returned `#t`.

Example

```
(filter even? '(1 2 3 4 5)) => '(2 4)
(filter (lambda
          (x) (= (remainder x 3) 0))
        '(1 2 3 4 5 6 7 8 9 10)) => '(3 6 9)
```

Sadly, R5RS Scheme does not contain a `filter` function, but newer Schemes such as R6RS Scheme and Racket do, and it is simple to define our own version.

`foldl` and `foldr`

CAUTION: In Racket these are known as `foldl` and `foldr`, but in R6RS Scheme these are `fold-left` and `fold-right`. Unfortunately these are unavailable in R5RS Scheme, but once again these functions are trivial to define.

Definition

`foldl` and `foldr` `(foldl function init list1 ... listN)` and `(foldr function init list1 ... listN)` apply `function` to the elements of lists `list1` to `listN`. `function` accepts $N + 1$ arguments, where one of the arguments is `init`, which is the accumulated aggregate value. `foldl` starts from the left, while `foldr` starts from the right.

Examples

```
; Return the sum of all elements in the list
(foldl + 0 '(1 2 3 4))
```

The above code is the equivalent of performing the sum $0 + 1$ (where 0 is from the `init` parameter) and storing the result in `init`, then $1 + 2$, then $3 + 3$, then finally $6 + 4$, resulting in 10.

Examples

```
(foldl
  (lambda (x count)
    (if (even? x)
        (+ 1 count)
        count))
  0
  '(1 2 3 4 5 6 7 8 9 10))
```

The above code counts the number of even numbers between 1 and 10.

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Quicksort Demo

I will be giving a demo of Quicksort implemented in Scheme using the DrRacket IDE.

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Topics To Be Covered

- The Concept of State in Programming
- Mutation in Scheme
- Environments (used for evaluation purposes)